WAGE INCENTIVE PAYMENT FOR MULTIPLE

MACHINE ASSIGNMENTS

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TABLE OF CONTENTS

TABLE OF CONTENTS CONT'D

LIST OF TABLES

LIST OF FIGURES

WAGE INCENTIVE PAYMENT FOR MULTIPLE MACHINE ASSIGNMENTS

THE PROBLEM

The wage incentive method of compensating employees has been a principal factor in America's industrial and social progress. When paid by the wage incentive plan the worker is, in effect, in business for himself; the more he produces, the more he earns. By increasing his output within existing facilities, the employee lowers the overhead cost per unit of product, thereby permitting management to lower selling prices to attract more sales. The resulting increased demand creates more jobs and the cycle continues to the advantage of all.

The success of any wage incentive plan is largely dependent on the consistent fairness of the incentive employee's earnings. In the last few years, time study techniques have been refined to the point that fair wage incentive rates are possible for most pure manual or single machine operations. In the case of multiple machine assignments, however, considerable trouble has been experienced in establishing fair incentive rates.

Unlike pure manual or single machine operations, multiple machine assignments generally involve ever-changing work loads. Some assignments entail considerable machine interference idleness and very limited unavoidable waiting time on the operator's part, while other assignments involve only limited machine interference idleness but significant unavoidable waiting time by the operator. As will be shown later, the multiple machine operator's unavoidable idle time, which provides a basis for wage incentive payment, is directly influenced by machine interference.

In his efforts to evaluate machine interference in establishing wage incentive rates, the time study man has often resorted to stop-watch timing. The multitude of irregular and spasmodic happenings in multiple machine assignments, however, usually provoke so much confusion that the job of timing machine interference becomes a hopeless task.

REVIEW OF THE LITERATURE

A thorough bibliographical search involving 15 articles failed to produce what the writer would consider a correct solution to the general problem of machine interference. Most of the articles studied dealt with specific conditions common to only a very limited number of cases of multiple machine assignment. The formulas and tables presented in these articles were either based on actual timing of interference, subject to the errors of human judgement, or entailed mathematical assumptions which are highly debatable.

Of the articles studied, Pinkerton's[1] theory for the solution of the machine interference problem appears to be the most accurate. Although this article treats of the laws of probability, which, according to the experiments to be described, provide a valid basis for solution of the problem, the fact that interference causes interference has been

[1]Pinkerton, D.W., "When Pieceworker Runs Several Machines", American Machinist, Vol. 76, No. 26, July 8, 1932, pp. 816-8.

overlooked. This apparently accounts for the great differences between the interference experiment results to be described and interference values computed using Pinkerton's theory.

SUMMARY

A mathematical solution of the problem of machine interference has made it possible for the writer to develop a wage incentive plan for multiple machine assignments. The plan is designed to cope with the widely varying characteristics inherent in this type of work. It is designed to provide for consistently fair incentive pay for assignments of 4 to 100 machines tended by one operator, regardless of the combinations of the products manufactured or the sizes of the work loads.

The multiple machine wage incentive plan provides for compensating the operator in two ways. For a given assignment, the operator's worked time earnings are first computed on the basis of the production counts and respective rates for the products manufactured. Second, the unavoidable waiting time that would have been experienced by the average operator due to the work load is figured. This idleness is compensated for on the basis of the employee's guaranteed hourly pay.

The research and development of the wage incentive plan to be described can be divided into three major steps: (1) the development of the mathematical solution to the machine interference problem, (2) the development, construction and testing of the machine interference computer which was made for the purpose of testing the validity of the mathematical methods of evaluating machine interference, and (3) the development of the wage incentive plan for multiple machine assignments,

based on the mathematical method of determining machine interference. The material to be presented is arranged in this order.

DEFINITION OF TERMS USED

Guaranteed base rate is the rate of hourly pay granted the worker who is on non-incentive work. It is also the minimum guaranteed hourly earnings of the incentive operator and is the rate at which he is paid during unavoidable idle time.

Load refers to the percentage of total time that the operator will spend in performing the necessary servicing duties in behalf of a given machine, assuming the machine is to be tended individually and the operator is to exert normal productive effort when performing the servicing duties.

Machine interference is the non-productive time a machine experiences when the operator is not available for the necessary servicing duties, but is servicing another machine in the assignment.

Multiple machine assignments refer to jobs which require that the operator tend two or more machines.

Normal refers to the rate of movement on the operator's part which is indicative of natural body movements free of waste motion. It assumes the operator is trained and qualified to perform the job in question.

Over-assigned refers to situations in which the aggregate work load of the individual machines assigned to the operator is so great that, regardless of the operator's productive effort, there will always be at least one machine awaiting servicing by the operator.

Productive effort refers to the actual rate of movement exerted by the operator, stated in terms of normal. For example, when 100% denotes normal productive effort, a productive effort of 120% would represent a rate of productive movement 20% faster than normal.

Servicing refers to the act of performing some necessary duty in behalf of the machine, on the part of the operator. The machine may be running or idle during the servicing.

Shut down means non-productive time on the part of the machine. It may result from regular servicing or machine interference.

Unavoidable idle (or waiting) time refers to those occasions during which all machines assigned to the operator are producing simultaneously. During such times the operator is unavoidably idle.

Under-assigned is the converse of "over-assigned". It refers to situations in which the aggregate work load of the machines assigned to the operator is limited to the extent that the operator experiences unavoidable idle time.

Work load is the aggregate of the loads of all machines to be assigned to one operator for a given period of time.

Worked time refers to the ratio or percentage of elapsed time on the job that the operator is engaged in the necessary servicing duties.

MACHINE INTERFERENCE

Machine interference occurs when one or more machines are non-productive because, having shut down in the need of servicing, they stand idle because the operator is tending another machine. In multiple machine assignments machine interference is generally inevitable because the running cycles of the machines cannot be coordinated. Simultaneous chance shut downs of machines are the rule rather than the exception.

The effects of machine interference are clearly apparent in the textile industry. When, for instance, an operator is engaged in servicing a given loom, one or more of the remaining looms in the assignment may chance to shut down. There will be occasions when many looms are idle at the same time as well as occasions when all looms are producing simultaneously.

Most multiple machine assignments entail several interference provoking features. As described above, the occurrence of shut down time on the part of each machine in a multiple machine assignment is generally a matter of chance. Furthermore, the durations of the shut down occasions usually vary considerably due to the operator's productive effort as well as the nature of the servicing requirements.

In spite of the fact that machine interference is unpredictable and variable, it has been found in the experiments to be described that it can be very closely measured for a great variety of conditions. Two mathematical solutions of the machine interference problem will be developed in this thesis. Both solutions are based on the laws of probability and, although quite different in their approach to the problem, provide identical results for a given set of conditions.

MATHEMATICAL SOLUTION OF MACHINE INTERFERENCE PROBLEMS

One of the laws of probability states, in effect, that when an event is based on chance as pertaining to each of several participants acting together, the various possible combinations of occurrence of the event will be distributed according to the terms of a binomial expansion, i.e., $(d+r)^n$[1]. For example, suppose it is desired to determine the probabilities of each of the possible combinations of occurrence of the ace when rolling three dice together, i.e., the chance of all three dice showing aces together, the chance of rolling two aces and a non-ace together, etc. This problem may be solved as follows:

Let d = the probability of occurrence of the ace for each die; $d = 1/6$.

Let r = the probability of failure of occurrence of the ace for each die; $r = 5/6$.

Let n = the number of dice; $n = 3$.

For these conditions, $(d+r)^n$ becomes $(1/6 + 5/6)^3$.

The expansion of $(1/6 + 5/6)^3$ can be arranged as follows:

No. Aces Showing	Binomial[2] Coefficients		(d)		(r)	Probability
3	1	x	$(1/6)^3$			1/216
2	3	x	$(1/6)^2$	x	$(5/6)^1$	15/216
1	3	x	$(1/6)^1$	x	$(5/6)^2$	75/216
0	1			x	$(5/6)^3$	125/216
						216/216

Here it can be seen that on the average during 216 rolls of three dice, three aces will turn up once, two aces and a non-ace will occur 15 times, one ace and two non-aces will show 75 times, and three non-aces will turn up 125 times.

The problem of computing interference for a given number of machines can be handled in the same manner as the dice problem, except that

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[1] $(d+r)^n = d^n + \frac{n}{1!} d^{n-1} r + \frac{n(n-1)d^{n-2}r^2}{2!} + \ldots \text{ etc} \ldots + r^n$

[2] The terms immediately preceeding d in the binomial expansion are known as "binomial coefficients", and follow a definite pattern, shown in Table I.

interference waiting time for each probability of machine shut down must be factored in. For example, assume three machines chance to shut down during the same interval of time. While one machine is being tended by the operator, the other two machines must wait. The interference idleness inherent in the probability of three machines being down at the same time would therefore be that probability multiplied by the two consequent waits. The sum of the various probabilities of interference idleness is divided by the number of machines tended to arrive at the average interference idleness per machine.

A brief description of the application of this interference evaluation method to a simple problem will now be taken up. To make this presentation clear, the problem selected involves the determination of interference after a period of operation; normally the interference allowance would be predetermined for purposes of wage incentive payment.

Problem: Assume one operator tends six semi-automatic machines. A production count at the end of the day reveals that the average non-producing time per machine was 20% of the time the machines were operated. It is desired to determine the average percentage interference and the average percentage servicing time for each of the machines tended.

Solution:

Let d = the average ratio of down time, i.e., non-producing time, to total operating time for each machine; $d = 1/5$.

Let r = the average ratio of producing time to total operating time for each machine; $r = 4/5$.

Let n = the number of machines tended by the operator; $n = 6$.

Substituting, $(d + r)^n$ becomes $(1/5 + 4/5)^6$.

The expansion of $(1/5 + 4/5)^6$ follows:

No. Machs. Down Together	Coeff. Sums		(d)		(r)		Probability	Waits	Interference
6	1	x	$(1/5)^6$			=	1/15625	5	5/15625
5	6	x	$(1/5)^5$	x	$(4/5)^1$	=	24/15625	4	96/15625
4	15	x	$(1/5)^4$	x	$(4/5)^2$	=	240/15625	3	720/15625
3	20	x	$(1/5)^3$	x	$(4/5)^3$	=	1280/15625	2	2560/15625
2	15	x	$(1/5)^2$	x	$(4/5)^4$	=	3840/15625	1	3840/15625
1	6	x	$(1/5)^1$	x	$(4/5)^5$	=	6144/15625	0	
0	1			x	$(4/5)^6$	=	4096/15625	0	
					Totals		15625/15625		7221/15625
							100%		.462 machine

Since there are six machines in the group, the average interference per machine would be .462/6 or 7.7%. Since down time, d, is equal to average servicing time plus average interference time, the average servicing time per machine would be 20.0%- 7.7% or 12.3%.

It is interesting to note that, since total down time, d, is equal to regular servicing time plus average interference time, interference causes interference. Interference caused by regular servicing would result when, for instance, three machines out of a group shut down at the same time. Two machines must wait while the other is being serviced. Later, moreover, while the operator services one of the remaining two idle machines, the other must continue to wait. This waiting time of the third machine would be attributed to the prior interference of the second machine, and it would therefore be a case of interference causing interference. Meanwhile, if any of the other machines in the group chances to shut down while the operator is engaged in the servicing described above, there would be another case of interference causing interference.

The application of this mathematical method of interference determination is much more difficult when predetermining interference rather than determining interference after the actual production loss is known, as was done in the foregoing case. This can best be shown by means of a problem. Problem: Assume one operator is to be assigned six machines, each requiring 15.5 minutes normal servicing time if tended individually, including average walking requirements from a central point with reference to all six machines, and 84.5 minutes automatic running time for every 100 minutes of operation. It is desired to determine the average interference idleness per machine when the operator performs the servicing duties with an average of 120% productive effort.

Solution: Since machine down time, d, consists of both servicing time and interference time, and the interference is the unknown, it is apparent that interference must be estimated for use in d. If, then, the interference computed by the expansion of $(d+r)^n$ agrees with the estimated interference used in its determination, the true interference is known; otherwise new interference estimates will have to be made and the problem re-worked until agreement is reached.

In solving the problem in hand, it is first necessary to determine what the percentage actual servicing time to total time would be if the machines were tended individually. Applying the operator's expected 120% productive effort, this would be:

$$(15.5 \text{ min.}/120\%)/(15.5 \text{ min.}/120\% + 84.5 \text{ min.}) \text{ or } 13.3\%$$

It is now necessary to estimate the interference inherent in an assignment of six machines, each of which would be serviced 13.3% of the total time if tended on an individual basis. Let 7.7% be the estimated interference per machine. If each of the machines will be non-productive an average of 7.7% of the total time due to interference when tended in multiple, the percentage servicing time to total time for each machine will be:

13.3% (100%-7.7%) or 12.3%

Down time, d, will therefore be 12.3% servicing time + 7.7% interference time or 20%. Referring to the previous problem, it can be seen that the 7.7% interference conveniently assumed for the above conditions to give d = 20% (or 1/5), does result in 7.7% interference per machine and is therefore the true interference for the conditions stated.

Briefly, the steps necessary for computing average interference per machine using the method just described are as follows:

1. Determine the number of machines to be assigned to the operator.
2. Determine the average percentage of overall operating time that each machine will require servicing, assuming each machine is to be tended individually by the operator who will work from a point involving average walking requirements per machine necessary when all machines are tended together. It will be necessary to take into account the operator's expected productive effort when arriving at the average servicing time per machine.
3. Estimate the average machine interference idleness, in percentage of overall operating time, to be encountered by each of the machines when they are tended together.
4. Add the estimated average interference per machine to the

estimated average servicing time per machine on an individual attention basis. This value represents the average percentage down time, d, per machine when n machines are tended together.

5. Determine the average running time, r, per machine when n machines are tended together; $r = 1-d$.
6. Compute the various probabilities of shut down of the machines by expanding $(d + r)^n$.
7. Multiply the various shut down probabilities by the consequent numbers of interference waits. For example, when four machines shut down simultaneously, three of the machines must wait due to machine interference. The probability of four machines being shut down at the same time must therefore be multiplied by three.
8. Total the interference values for each of the probabilities and divide the total interference by the number of machines assigned to arrive at the average percentage of overall operating time that each machine will be idle when n machines are tended together.
9. Compare the computed interference with the interference estimated in step 3. If there is a significant difference between these two values, a new estimate must be made and the problem re-worked as described in steps 3-9.

Although the experiments to be described gave proof of the validity of the foregoing method of computing machine interference for the many conditions tested, this solution of the interference problem has practical limitations in that a considerable amount of time would be required to calculate the true interference for each multiple machine assignment. It was therefore decided to construct interference curves which would permit rapid solution of machine interference problems. Even for purposes of curve construction, however, the foregoing method of determining machine interference would involve a prohibitive amount of time. Further study revealed that an alternate application of the probability theorems provides an equally valid but much quicker method of computing machine interference. This method was used in constructing the machine interference curves of Figure 2.

AN ALTERNATE MATHEMATICAL SOLUTION OF MACHINE INTERFERENCE PROBLEMS

As can be noted in the application of the binomial theorem in determining probabilities, the probability of two or more independent events happening simultaneously is the number of possible ways that the simultaneous occurrence can happen multiplied by the product of the individual probabilities of the event happening to each participant. This principle provides the basis of an alternate solution of the problem of machine interference. The formula is developed as follows:

Let the original definitions of n,d and r stand.

Let s = the average ratio of total operating time each machine will be non-productive due to servicing when n machines are tended together.

Let i = the average ratio of total operating time each machine will be non-productive due to machine interference when n machines are tended together.

From the foregoing, it follows that d, the average ratio of total operating time each machine will be non-productive, is equal to $s \neq i$.

Applying the aforementioned law of probability when one operator tends n machines, the probability that all n machines will be running simultaneously at a given moment is $(r)^n$.

Consequently, the probability that one or more machines will be non-productive at a given moment will be $1-(r)^n$ or $1-(1-d)^n$.

The average ratio of servicing time, s, to total operating time for each machine when one operator tends n machines will therefore be $\frac{1-(1-d)^n}{n}$.

By assuming various d and n values it was possible to accurately determine the corresponding s values through the use of the expression $s = \frac{1-(1-d)^n}{n}$. Then the respective i values were determined by substituting in the expression $i = d-s$.

For purposes of wage incentive payment for multiple machine assignments it is necessary to know the percentage servicing time to total operating time required for each product, assuming the machine with which the product is to be made is tended individually. The reasons for this will be apparent later. It was therefore necessary to divide the computed s values, based on multiple operation, by unity minus the respective i value; $S_{E_a} = \frac{s}{1-i}$, where S_{E_a} represents the average actual percentage servicing time to total operating time for each of n machines to be tended by one operator, assuming each machine is tended individually with E productive effort. Finally, the interference curves of Figure 2 were prepared on the basis of the foregoing procedure. Given the average servicing requirements per machine, S_{E_a}, for a group of n machines to be tended by one operator, it is a simple matter to determine, through the use of Figure 2, the average percentage interference idleness, i, to be experienced by each of the machines when tended together.

THE MACHINE INTERFERENCE CURVES

The machine interference curves of Figure 2 provide a rapid means of closely approximating the average loss of time due to machine interference for a given set of conditions. The use of Figure 2 can best be described by employing it in the solution of a simple problem.
Problem: Assume one operator is to be assigned ten machines and it is desired to determine the average percent interference idleness per machine when the operator exerts 125% productive effort. If tended individually with normal productive effort from a point involving average walking necessary when all ten machines are tended together, three of the machines would require 14% normal servicing time each, four would require 8% normal servicing time each and three would require 20% normal servicing time each.

Solution: The total normal load for the assignment is 3 x 14% + 4 x 8% + 3 x 20% or 134%. The average normal load per machine for each of the ten machines is 134%/10 = 13.4%. If each of the machines were tended individually with 125% productive effort, however, the average actual servicing time per machine would be (13.4%/125%)/(13.4%/125% + 86.5%) = 11.0%. The average interference per machine for each of the 10 machines can be quickly determined from Figure 2 in the following manner.

1. Locate 10 machines on the left vertical "Number of Machines Tended By One Operator" scale.

2. Project horizontally to the 11% "Ave.% Servicing Time Per Machine" curve.
3. At the point of intersection, drop vertically to the "Ave. % Interference Per Machine" scale; the answer for the conditions of this problem is 15% average interference per machine for each of the ten machines.

The interference curves of Figure 2 were used in developing the unavoidable waiting time curves of Figures 8 and 9 , which provide for the flexible feature of the wage incentive plan to be described.

ANALYSIS OF THE MATHEMATICAL SOLUTIONS OF THE MACHINE INTERFERENCE PROBLEM

There are many questions which might be raised regarding the practical validity of the two mathematical methods of computing interference just described. The binomial theorem, which is the basis of the first method, has been proved mathematically valid for figuring probabilities of uniformity and absolute chance. The occurrence of shut down time in multiple machine assignments, however, is seldom based on absolute chance, even though the running cycles of the machines cannot usually be coordinated. Furthermore, the durations of servicing requirements of semi-automatic machines are usually variable. Also, it is exceptional when one operator is assigned several machines having uniform total servicing time requirements. Specifically, in deciding on the reliability of the binomial expansion method of determining machine interference, the question of whether the expansion of $(d+r)^n$, factored with average waiting time yields the true interference for multiple machine operation conditions of semi-chance, when d and r are variable and are employed in the expansion as averages, must be answered.

The alternate solution to the machine interference problem is also subject to question.[4] For example, if the individual ratios of running time, r, to total time for each of n machines tended in multiple are significantly different, the ratio of total time that all n machines will be running simultaneously will not be $(\Sigma r/n)^n$ or $(r_{ave})^n$ as employed in the alternate method of computing machine interference; the ratio would be $r_1 \times r_2 \times \ldots \times r_n$. It is believed, however, that when machines having different servicing requirements are tended in multiple, the ratios of down time, d, to total operating time for the various machines are in close enough agreement to make $(r_{ave})^n$ a valid basis for computing machine interference with sufficient accuracy for wage incentive purposes. Therefore, as in the case of the machine interference solution employing the binomial expansion, the question of validity of the use of averages in the alternate method seems to be the deciding factor in determining its accuracy.

In seeking to determine the practical validity of the mathematical solutions of the problem of machine interference it was recognized that two requirements were essential in order to reach a valid decision. First, a wide variety of typical multiple machine assignments would have to be studied and the machine interference losses actually timed.

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[4] The expression $(r_{ave})^n$ is not equivalent to $r_1 \times r_2 \times r_3 \times \ldots$ etc.. $\times\, r_n$ when the individual r's differ. To illustrate, assume $r_1 = 1/4$, $r_2 = 1/4$, $r_3 = 1/3$, and $r_4 = 1/2$. For these conditions $r_{ave} = (1/4+1/4+1/3+1/2)/4$ or $1/3$ and $(r_{ave})^n = (1/3)^4$ or $1/81$. On the other hand, $r_1 \times r_2 \times r_3 \times r_4 = 1/4 \times 1/4 \times 1/3 \times 1/2$ or $1/96$.

Second, the timing would have to be done mechanically, insuring against human error. The interference computer shown in Figures 3,4, and 5 was developed to fulfill these requirements.

THE INTERFERENCE COMPUTER

The interference computer[5] was developed for the purpose of determining the practical validity of the mathematical methods of evaluating interference. The equipment simulates the semi-automatic machine characteristics which influence machine interference, causes machine interference, and then accurately measures the same.

Referring to Figure 5, the equipment consists of three principal components: (1) a group of ten machines, A, placed in a circle, (2) an operator, B, which moves about in a circle, servicing machines which shut down and (3) a timing and revolution counting apparatus, N and O, used to provide the data necessary for evaluating the servicing time and interference time for the experiments.

Each of the ten machine units consists of a transparent plastic disc, C, placed on a round table, D, and centered by a slipfit over a shaft, the principle employed in the conventional phonograph table and record arrangement. Each table revolves at a slow, constant speed of 22.5 revolutions per hour by means of a V-belt drive. Positioned tangent to each disc, and equidistant from the center of the circle of the ten units, are detectors, E, which pivot vertically to contact the upper surfaces

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[5] Jones, W. Dale, "Mathematical and Experimental Calculation of Machine Interference Time", The Research Engineer, Georgia Institute of Technology, January, 1949 pp. 9,10,20-23.

of the discs at points near their circumferences. Rectangular servicing blocks, F, of any desired length and number are fastened to the top surface of each disc, flush with the outer edge.

These block-bearing discs revolve with the table upon which they rest. The detector, when contacted by a servicing block, first rises slightly and then catches or stops the movement of the block and consequently, the disc upon which the block is mounted. In the meantime, the round table which supports the disc continues to revolve at a slow constant rate. Cessation of movement of a disc represents the corresponding shut down on the part of the machine.

The required servicing of the individual machines is performed by an operator which revolves in the path of the detectors of the machines. This operator consists of a narrow arm extending from a platform, G, located in the center of the circle of detectors. The operator's platform, like the individual discs, rests on a table which revolves at a constant rate of ten revolutions per minute, except when interrupted by the machines.

As was mentioned previously, when a block passes beneath the detector, it causes the detector to rise, after which the block catches on the detector pawl, H, causing the disc to cease revolving. This is interference idleness, since the machine is waiting for the operator. The magnitude of this interference idleness for each machine over the test period is later determined by subtracting the number of actual revolutions of each disc from the number of revolutions theoretically possible had there been no interference. Having been elevated by the block, the detector is in the path of the operator.

The operator, upon approaching the idle machine, raises the detector pivot arm, I, slightly in order to release the pawl from the block and to permit the disc to resume moving in conjunction with its table. After the detector arm is raised and disengaged from the block, it is momentarily released by the operator so that the detector pawl thereafter rests on top of the now moving block. As this begins, the operator comes in contact with the end flange, J, of the detector, and because of the elevated position of the detector, the operator must remain stationary, servicing the machine, until the block completes passing under the detector, after which the detector drops to its normal position in contact with the disc surface. In resuming its normal position, the detector breaks contact with the operator arm, permitting the operator to proceed to any of the other machines which might have chanced to shut down.

The equipment also includes a mechanism which causes variability and unpredictability in the movement of the ten block-bearing discs. Two interrupters, K, which rise from beneath each disc to interfere with the equally spaced pins, L, are actuated by five constantly revolving cams, M, of different lengths, moving at different speeds. The irregular interruption caused by this arrangement introduces the variability and unpredictability present in most semi-automatic machine running cycles when one operator tends several machines. Over the test period, however, each disc is interrupted for an equal length of time.

The servicing time and interference time data are provided by the electric timer, N, and the revolution counters, O. The electric timer is controlled by the servicing blocks and the detectors, recording time only when its circuit is completed by at least one servicing block and detector contact. In other words, the timer is in operation only when one or more of the machines is shut down due to interference or due to servicing by the operator. Since the operator is either approaching or servicing a machine when the electric timer is in operation, the time accumulated on the timer during the test represents the operator's total worked time for the period. The difference between the test time and the worked time represents the operator's unavoidable waiting time due to the limitations of the work load.

TESTING WITH THE INTERFERENCE COMPUTER

Sixty-two tests were conducted with the machine interference computer to determine machine interference versus servicing time for various work loads involving four thru ten machines. The tests involved uniform conditions when each machine had the same degree of servicing requirements as well as non-uniform conditions involving different degrees of servicing time for the various machines. The tests were conducted in the following manner:

1. Servicing blocks were fastened to the circumference of each of the discs of the machines represented in the test. Circuit connection with the electric timer was made for each servicing block by connecting the leads to the blocks. When the tests involved uniformity in servicing requirements of the machines, it was necessary to place the servicing blocks at the same respective locations on each disc with reference to the interruption pins and the disc circumference. When the

tests involved non-uniform servicing requirements of the machines, the servicing blocks were fastened to the various discs in a non-uniform manner.

2. The readings from the revolution counters of the machines and the electric timer were carefully noted and posted.
3. The interference computer was then started and the starting time posted.
4. The computer was permitted to run continuously for 30 minutes.
5. At the end of the 30 minute test period the interference computer was stopped and readings were taken from the revolution counters of the machines and the electric timer.
6. The average revolutions per disc for each machine was determined. This was done by subtracting the beginning revolution counter reading from the end reading for each machine and then averaging the results.
7. The average percent machine interference idleness per machine was evaluated. This was done by subtracting, for each machine, the actual revolutions per disc from the pre-determined average revolutions per disc possible in the absence of machine interference, totaling and averaging, and then dividing the average by the non-interference revolution figure.
8. The average percent servicing time per machine was determined. This was done by dividing the elapsed worked time minutes as noted from the electric timer by 30 minutes, the duration of the test, and then dividing by the number of machines participating in the test.

9. The average percent servicing time per machine on an individual attention basis was determined. This was done by dividing the average servicing time per machine in multiple operation, as determined above, by unity minus the average percent interference per machine.

10. The average percent servicing time per machine and average percent interference per machine were posted to Figure 6, a graph consisting of mathematically determined machine interference curves for groups of 4 to 10 machines.

ANALYSIS OF THE INTERFERENCE COMPUTER TEST RESULTS

The consistently close agreement between the interference computer test results and the corresponding values secured via the mathematical methods developed in this thesis is evidenced by Figures 6 and 7 and Table II. Since Pinkerton's[6] solution to the machine interference appeared the soundest of the various theories studied, it was decided to include interference values calculated by his method in the comparisons of Figure 7 and Table II.

The relative accuracy of the Pinkerton and Jones methods of mathematically predicting machine interference for the conditions studied are shown as "% Deviation From Actual" in Table II. It must be kept in mind that these figures do not denote the percentage error of the interference allowances. They represent the differences in units of percent between the mathematical allowances and the true interference percentages. The average actual interference for the sixty-two tests was 12.1%.

[6] Pinkerton, D. W. loc. cit.

The average interference allowance applying to these conditions using the Jones solution was 12.4%. The average interference allowance using the Pinkerton solution was 4.2%. As was stated previously, Pinkerton's solution makes no provision for the fact that interference causes interference. This apparently accounts for the great differences between the actual interference values and the allowances computed via his method.

The extreme probabilities of the expansion of $(d+r)^n$, such as 6,7 or 8 machines shutting down simultaneously in the tests involving 8 machines, did not occur during the tests. This was apparently due to the fact that the occurrence of shut down by the machines studied was not a matter of absolute chance. The same difference between the theoretical probabilities of $(d+r)^n$ and actual occurrence of shut down will be found in most multiple machine assignments. Because, however, of the negligible weights of the extreme probabilities based on absolute chance, their absence has no significant effect on the theoretical interference values determined via the expansion of $(d+r)^n$ or the alternate method described.

There appears to be but one exception for which the foregoing mathematical methods of computing interference do not give representative results for the conditions studied; this being the case where the running cycles of the machines are so uniform and the servicing requirements so limited that the operator can coordinate the running dispositions of the machines as a means of minimizing interference. During the preliminary tests involving two and three machines, this fact became apparent. Tests involving these conditions were therefore discontinued.

On the basis of the consistent close agreement between the actual interference values and the interference curves for groups of 4-10 machines in Figure 6, and a statement from Mr. B.D. McAuley, Supervising Engineer, Stevenson & Kellogg, Ltd., Toronto, Ontario, the interference curves have been extended to include assignments of 100 machines to be tended by one operator. In his February 2, 1949 letter to the author, Mr. McAuley said:

> "In the few instances we have had where it was necessary to determine machine interference on assignments ranging from 80 to 100 machines, it would have been a formidable task to determine the interference by time study. We have used your interference tables and have found them to give very satisfactory results. The proof of the results is that the relative operators' earnings on assignments were in line with their observed effort ratings on the manufacturing floor."

THE WAGE INCENTIVE PLAN

With the interference curves of Figure 2 in hand it is possible to provide consistently fair, equitable pay to multiple machine operators in spite of the ever-changing work loads characteristic of this type of work.

For wage incentive rate setting purposes, multiple machine assignments may be divided into two classes: (1) assignments for which the aggregate servicing requirements are less than a full load on the part of the operator and (2) assignments in which the operator is over-assigned, so that regardless of his productive effort, there is always at least one machine idle, awaiting servicing. The problem of wage incentive payment for assignments falling in the second category should not be difficult. By setting the piece rates on the basis of normal walking and servicing time per unit and ignoring automatic running time, the operator's earnings for a given assignment can be determined by multiplying the totals of the various products manufactured by their respective rates. Assuming all other factors affecting the operator's incentive rate have been handled correctly, the operator's earnings, when figured by the above method, would be directly proportional to his productive effort.

Unfortunately for the rate setter, it is seldom feasible to over-assign the operator. It is generally better to minimize machine interference idleness by limiting the operator's work load to the extent that he spends a significant portion of the time waiting in readiness for servicing requirements. Due to chance, these unavoidable waits are irregular, widely varying in duration, and generally beyond the control of the operator. It therefore follows that a means of accurately evaluating and compensating for this unavoidable idle time is necessary if the operator is to be compensated by incentive rates based on servicing and walking time only, as previously described.

To illustrate the problem of wage incentive payment for cases of varying work loads, let us assume Mr. Average Operator is over-assigned during the morning hours of a given day and under-assigned during the afternoon hours. His unavoidable idle time is zero during the four hours of the morning but assume it is 20 percent during the four afternoon hours. According to the wage incentive plan to be described, the operator's pay for the eight hours would be determined in two steps. First, the totals of the various products manufactured during the day would be multiplied by their respective rates to arrive at the operator's earnings during his actual working time. Second, the operator's unavoidable idle time (20% x 4 hrs.) would be evaluated and compensated for at guaranteed base pay or something greater, depending on company policy. From day to day, each operator on multiple machine assignments would be paid in the above manner, i.e., each operator would be paid for his total output as well as a computed unavoidable waiting time allowance for the individual assignment, figured on the basis of the average productive efficiency of all operators included in the wage incentive plan.

As will be demonstrated in the problems to follow, the pay clerk's role in this wage incentive plan can be comparatively simple provided two things are in hand. First, the pay clerk must have a set of wage incentive rates for all the products manufactured in multiple. As previously described, these rates are to be based on walking and servicing time only. Each rate, however, would consist of two parts. There would be one figure representing the standard time or pay allowed per unit, per thousand units, etc. of the product. In addition, there would be a percentage figure denoting the percentage of total time that would be required of the operator if he should individually tend, with normal productive effort, the machine on which the product is to be made. These individual loads would be totaled for a given assignment to arrive at the total normal work load. Second, the pay clerk must have a means of rapidly determining the extent of unavoidable idle time in a given assignment resulting from the total normal work load, figured on the basis of the expected average incentive productive effort of all operators covered by the plan. Figures 8 and 9 have been developed for this purpose. Before taking up the application of the unavoidable waiting time curves of Figures 8 and 9, the formula employed in their construction will be developed.

> Let S_d = the normal servicing walking and fatigue time in minutes per unit of product, which will normally be performed while the machine is non-productive. In evaluating S_d, assume the machine with which the product is to be manufactured will be individually tended by the operator who will work from a point involving average walking requirements per machine when all n machines are tended together.
>
> Let Sp = the normal servicing, walking and fatigue time in minutes per unit of product, which will normally be performed while the machine is producing. As in the case of S_d, assume the machine is to be individually tended.
>
> Let R = the average automatic producing time in minutes per unit of product.
>
> Let E = the operator's productive effort (or efficiency) when performing the necessary walking and servicing duties. The base of E is 100%, or unity.

If the operator's productive effort, E, is 100% (or normal) when tending one machine individually, the ratio of servicing time to total operating time will be

$$(S_d + S_p)/(S_d + R) \text{ or } S$$

When, however, the operator's productive effort is something other than normal, the percentage servicing time, S_E, to total operating time becomes:

$$(S/E)/\left[S/E + (100\% - S)\right]$$

When the operator tends n machines, machine interference idleness becomes a factor in the operating disposition of each machine. The percentage servicing time to total time, S_E, for each machine, figured on an individual attention basis, therefore becomes S_E (100%-i) when n machines are tended in multiple, where i represents the average percent of total operating time each machine will be idle due to machine interference. The operator's percentage unavoidable idle time, U, when tending n machines having individual servicing requirements of S_{E_1}, S_{E_2}, ... etc..S_{E_n} is therefore

$$100\% - (S_{E_1} + S_{E_2} + \ldots etc.. + S_{E_n})(100\% - i)$$

where i is based on

$$\frac{S_{E_1} + S_{E_2} + \ldots etc.. + S_{E_n}}{n} \quad \text{or } S_{E_a}$$

and is secured from the interference curves of Figure 2. Of course, when U is zero or negative, the operator is over-assigned, i.e., the operator has no unavoidable idle time for the assignment in question.

Figures 8 and 9 were prepared for the purpose of rapidly determining the operator's unavoidable idle time, U, for various circumstances. Given the servicing requirements, S, for each of a group of n machines to be tended by an operator working at E productive efficiency, it is a simple matter to pre-determine from Figure 8 the extent of unavoidable idle time to be encountered by the operator when tending the machines. Figure 9 goes a step farther than Figure 8 in that it assumes a productive effort, E, of 125%. With Figure 9 in hand, the pay clerk can quickly compute unavoidable idle time allowances by merely determining the aggregate S, i.e., the total normal work load, for a given assignment of n machines and then referring to the curves. Since the average productive efficiency of multiple machine operators when on incentive is in the neighborhood of 125% during the necessary servicing duties, Figure 9 is recommended for general use in evaluating unavoidable idle time allowances.

Case Problem No. 1. *The wage incentive rates for four products* and the methods of computing same are as follows:

Product	Sd	Sp	R	Norm.Work Load (Sd+Sp)/(Sd+R)	Incentive Standard (Sd + Sp)(1000)/60
A	11.06	3.72	97.23	13.6%	.246 hr./1000:(13.6%)
B	10.91	4.16	80.79	16.4%	.251 hr./1000:(16.4%)
C	8.44	2.06	83.97	11.4%	.175 hr./1000:(11.4%)
D	4.10	1.95	92.68	6.3%	.108 hr./1000:(6.3%)

Note: The values for S_d, S_p and R are expressed in terms of minutes per thousand units produced. Allowances for personal time and minor interruptions are omitted for purposes of clarity.

Determine the operator's unavoidable idle time, U, when he is tending 3 machines producing A, 3 machines producing B, and 5 machines producing D, when exerting (a) 125% productive effort during servicing (b) 150% productive effort during servicing.

Solution. The total normal servicing load of the assignment on an individual attention basis is 3 machines x 13.6% + 3 machines x 16.4% + 5 machines x 6.3% or 121.5%. The average normal percentage servicing time, S, per machine for each of the 11 machines is 121.5%/11 or 11.05%. The average percentage actual servicing requirements, S_{E_a}, per machine if tended individually, when the operator exerts 125% productive effort, would be

$$(11.05\%/125\%)/(11.05\%/125\% + 88.95\%) \text{ or } 9.05\%$$

When the operator exerts 150% productive effort, S_{E_a} would be

$$(11.05\%/150\%)/(11.05\%/150\% + 88.95\%) \text{ or } 7.65\%.$$

As previously developed, the operator's unavoidable idle time,

$$U = 100\% - (S_{E_1} + S_{E_2} + \text{..etc..} + S_{E_n})(100\%-i)$$

or

$$U = 100\% - n\,S_{E_a}(100\%-i).$$

At a productive effort, E, of 125% for the conditions of this problem,

$$U = 100\% - (11)(9.05\%)(100\%-10.8\%) \text{ or } 11.0\%.$$

At 150% productive effort,

$$U = 100\% - 11(7.65\%)(100\%-6\%) \text{ or } 21.0\%.$$

The interference values above were secured from Figure 2 on the basis of S_{E_a}.

Case Problem No. 2. An operator who receives $1.00 per hour base pay is to be assigned three machines producing B (see Problem No.1), four machines producing C and five machines producing D. Determine how much the operator will earn in eight hours when performing the servicing duties with 120% productive effort, if the company's policy is to grant 100% allowance for unavoidable idle time, U, figured on the basis of an average of 125% productive effort of all incentive operators (Figure 9).

<u>Solution</u>. The total normal servicing load for this assignment is 3 machines x 16.4% + 4 machines x 11.4% + 5 machines x 6.3% or 126.3%. Referring to Figure 9, the operator's unavoidable idle time, U, for a normal servicing load of 126.3% when exerting 125% productive effort, is approximately 9%. The pay clerk will therefore credit the operator with 9% x 8 hours x $1.00 or $.72 for the unavoidable idle time, U, for this assignment. It must be kept in mind that the operator's actual unavoidable idle time will not be 9% in this case. The 9% idleness is figured on the basis of 125% productive effort whereas the operator's actual productive effort is 120%. Regardless of the operator's actual productive effort, however, the allowance for U is to be based on 125% productive effort, the average for all employees included in the wage incentive plan.

The average percent normal servicing time per machine, on an individual attention basis, for each of the 12 machines having a total normal servicing load of 126.3%, is 126.3%/12 or 10.5%. The average actual servicing time per machine, S_{E_a}, at a productive effort, E, of 120% is

$$(10.5\%/120\%)/(10.5\%/120\% + 89.5\%) \text{ or } 8.9\%.$$

The total actual servicing load, $n\,S_{E_a}$, when the operator exerts 120% productive effort would be 12(8.9%) or 107%. Referring to Figure 8, the average unavoidable idle time, U, for a load of 107% is approximately 7%. The operator's earnings for the eight hours will therefore be

120% x 93% time worked x 8 hrs. on job x \$1.00/hr. + \$.72 for U, or \$9.64. This represents 120.5% of basic guaranteed earnings.

Case Problem No. 3. An operator who receives \$1.00 per hour base pay produces the following products during eight hours:

Period	Product	Incentive Rates	No. Machs.	Normal Work Load	U (Fig. 9)*
8:00	A	.246 hr./1000: (13.6%)	2	27.2%	
to	C	.175 hr./1000: (11.4%)	8	91.2%	
10:00	D	.108 hr./1000: (6.3%)	2	12.6%	
		Totals	12	131.0%	7%
10:00	A	.246 hr./1000: (13.6%)	2	27.2%	
to	C	.175 hr./1000: (11.4%)	8	91.2%	
12:00	B	.251 hr./1000: (16.4%)	2	32.8%	
		Totals	12	151.2%	0%
12:30	C	.175 hr./1000: (11.4%)	8	91.2%	
to	B	.251 hr./1000: (16.4%)	2	32.8%	
4:30		Totals	10	124.0%	10.5%

* Referring to Figure 9, at 125% productive effort, an assignment of 12 machines with a total normal servicing load of 131.0% results in 7% U; 12 machines with 151.2% load = 0% U; 10 machines with 124.0% load = 10.5% U.

The operator's total production for the day is 4,270 units of A, 7,830 units of B, 34,500 units of C and 2,500 units of D. Determine his total earnings for the day, granting 100% allowance for unavoidable idle time, U, figured on the basis of 125% productive effort (Figure 9).

Solution. The operator's earnings during the actual working time are \$1.00/1000 (.246 x 4,270 + .251 x 7,830 + .175 x 34,500 + .108 x 2,500) or \$9.32. His earnings for unavoidable idle time, U, due to the sizes of the work loads are \$1.00 (2 hrs. x 7% U + 2 hrs. x 0% U + 4 hrs. 10.5% U) or \$.56. His total earnings for the day are therefore his working time earnings of \$9.32 plus his unavoidable idle time earnings of

$.56 or $9.88. This represents 123.5% of basic earnings.

ANALYSIS OF THE WAGE INCENTIVE PLAN

The conventional wage incentive plan is designed to provide for compensating the operator in direct proportion to his output when the incentive standards are exceeded. The incentive standard represents the output per unit of time that can be expected when a qualified, trained operator is performing the task with normal productive effort and when the operator is utilizing the allowances for fatigue, personal time and minor unavoidable interruptions included in the standard. Most plans specify that the operator will be guaranteed his basic hourly pay rate for the actual hours spent on incentive work on days[7] for which his output falls below the incentive standards. Also, most plans specify that the operator shall be taken off-standard[8] during major unavoidable interruptions such as waiting for material, waiting for machine repairs, etc. During the off-standard time the operator is paid on the basis of his guaranteed hourly pay rate. These provisions are generally accepted as being a fair basis for wage incentive payment.

As was previously pointed out, the problem of wage incentive payment for multiple machine assignments can be alleviated considerably by

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[7] Sometimes the operator's incentive earnings are figured weekly or monthly.

[8] Assuming allowance for the major unavoidable interruption time is not included in the wage incentive standard, the operator would be penalized unless taken off-standard because his efficiency during time worked on incentive is computed by dividing his earned time, i.e., output multiplied by the standard time per unit, by the actual time worked on incentive.

over-assigning the operator so that, regardless of his productive effort, there will always be at least one machine awaiting servicing. Under such conditions there is no unavoidable idle time, U, due to limitations of the work load. The operator's earnings for a given assignment can therefore be computed by multiplying the totals of the various products manufactured by their respective incentive rates, based on normal servicing, walking and allowance time per unit. Of course major unavoidable interruptions would be compensated for separately as previously described.

In cases where the operator is under-assigned to the extent that he is unavoidably idle a significant portion of the time due to the lack of servicing requirements on the part of the machines, the wage incentive problem becomes complex. As has been previously described, it is proposed that the unavoidable idle time inherent in a given assignment be compensated for in the same manner as major unavoidable interruptions. Realizing that the unavoidable idle time for a given set of conditions depends on the operator's productive effort and that this is a variable, it is proposed that the average productive effort of all operators included in the wage incentive plan be used in computing unavoidable idle time allowances. A productive efficiency of 125% has been taken as the average efficiency of multiple machine operators during the performance of the necessary walking and servicing duties when working on incentive operations. Figure 9, which assumes a productive efficiency of 125% is therefore recommended as a rapid means of determining the unavoidable idle time allowance for various circumstances.

The probable earnings for multiple machine operators for various conditions are shown in Tables III, IV, V, and VI. In reviewing these tables it might be well to keep in mind that the unavoidable idle time, U, occasions in multiple machines are generally so distributed and limited in duration that they are of little value for fatigue recuperation on the part of the operator. Of course the greater the aggregate percentage of unavoidable idle time, U, for a given set of circumstances, the longer the duration of the individual U occasions.

For the sake of comparison, three of the many possible methods of compensating the operator for U are included in the tables; one method completely ignores U, another method provides for 100% allowance for U, while the third method grants 125% allowance for U. Certain conclusions can be drawn from the tables.

1. An under-assigned operator cannot increase his earnings in direct proportion to a given increase in productive efficiency, the reason being that there is not proportionately more work available during the time saved by the increased productive efficiency.
2. An operator who is over-assigned to the extent that, regardless of his productive effort, there will always be one machine awaiting servicing, will increase his earnings in direct proportion to his productive effort since there will always be work to do.
3. For a given total normal work load and productive efficiency, E, the greater the number of machines, the less the U.

In the author's opinion, the plan of granting 100% allowance for U is the fairest of the three possibilities compared because:

1. It is generally agreed that the operator should be paid at least his base pay rate for time spent during delays beyond his control.
2. An allowance of less than 100% for U might discourage incentive effort on the part of the operator due to the comparatively small increase in earnings for a given increase in output.
3. A more lenient allowance of 125% of base pay for U might also discourage incentive effort because of what might be regarded by the operator as sufficiently high earnings for a comparatively low productive effort. This condition of over-payment to multiple machine operators may, in turn, provoke dissatisfaction on the part of operators on manual and single machine operations regarding the matter of equity of earnings.
4. An allowance of 100% for U results in an over all earnings percentage of approximately 120% when the average 125% E operator's U is 23%. Assuming one-half of the 23% U is of sufficient duration to aid in fatigue recuperation, the 120% earnings may be regarded even more than fair to the operator.

THE MATHEMATICS OF THE WAGE INCENTIVE STANDARDS

Like all incentive plans, the success of the multiple machine wage incentive plan is dependent mainly on the fairness of the incentive standards. Certain important rules must be followed in gathering the basic stop-watch data necessary for establishing incentive rates for products to be manufactured in multiple. Strict adherence to these rules will enable the rate setter to compile, with a minimum number of time studies, standard data which can be used in synthesizing a comparatively great number of wage incentive rates. To facilitate an understanding of the significance of the rules for time study for multiple machine rate setting purposes, the general mathematical expression of the incentive rates will first be developed.

Let the definitions for S_d, S_p and R stand, as presented on page 28.

Let P = the allowance for personal time, in terms of percentage of the operator's elapsed time on the job.

Let D = the allowance for unavoidable minor interruptions, in terms of percent of the operator's elapsed time on the job.

The standard time in terms of hours per thousand units of product can be stated as follows:

$$\text{Hrs./1000 units} = \frac{(Sd + Sp)(1000)}{(100\% - P - D)(60)}$$

As was illustrated in Case Problem No. 3, on page 33, the normal work load for each product must accompany the regular incentive standard. The individual normal work loads for the various products manufactured in each assignment are totaled for the purpose of determining, through the use of Figure 9, how much unavoidable idle time allowance the operator should be granted.

Referring to Case Problem No. 1 on page 30, the normal work load, i.e., the ratio of operating time[9] the qualified and trained operator's services would be required if tending a given machine individually with normal productive effort, is designated as

$$(S_d + S_p)/(S_d + R).$$

In its most convenient form, the wage incentive rate for a given product, in terms of hours per thousand units and the percent normal work load for the product, as employed in Case Problem No. 3 on page 33, is expressed mathematically as

$$\frac{(\mathrm{Sd} + \mathrm{Sp})(1000)}{(100\% - P - D)\ 60} \ \vdots\ \frac{(\mathrm{Sd} + \mathrm{Sp})(100\%)}{S_d + R}.$$

The left-hand term in the above expression represents the amount of time, in hours per thousand units, that a qualified and trained operator would spend in walking to and servicing the machine on which the product is being fabricated, assuming the operator utilizes the allowances included in the incentive standard and exerts normal productive effort during the walking and servicing duties. The right-hand term represents the percentage of operating time the operator who is qualified and trained for the job would spend in walking to and servicing

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[9] The term "operating time", as used here, represents the time the operator actually spends on the job. Personal time and unavoidable minor interruption time are not included.

the machine with which the product is to be manufactured, assuming the machine is individually tended and the operator exerts normal productive effort during the walking and servicing duties.

TIME STUDY PROCEDURE FOR ESTABLISHING INCENTIVE STANDARDS

The time study procedure necessary for establishing incentive standards for the proposed wage incentive plan is far less involved than most conventional procedures in the case of multiple machine assignments. In the proposed procedure the time study man ignores the ever-confusing matter of machine interference, whereas the conventional practice is to make an attempt at the almost impossible task of timing machine interference. As has been previously described, the problem of machine interference in the proposed wage incentive plan is taken care of in the interference curves of Figure 2, which, in turn, are reflected in the unavoidable idle time curves of Figure 9.

Since the curves of Figure 9 are based on aggregates of individual normal work loads for machines tended in multiple, and take into consideration an estimated average of 125% productive efficiency of multiple machine operators while on incentive, the time study man's job is reduced to the task of determining the normal and allowance time for each of the products to be produced in multiple, assuming the machines are to be individually tended. The recommended rules for time studying multiple machine operations are as follows:

1. List on the timestudy observation sheet, each of the products being fabricated in the assignment. Note the number of machines making each product.
2. Record the time the study begins.
3. Using the repetitive stop-watch timing method, time and rate the operator as he performs the necessary walking and servicing duties. Rate the operator during the time study rather than after the study. Each time the operator performs a servicing duty, a symbol should be recorded beside the stop-watch time value to denote the nature of the service. For example "creel" can be designated with a "C" and "doff" can be denoted with a "D". Walking should be timed and rated separately, and can be abbreviated with the "W". Whenever possible, a subscript, denoting whether the machine approached or serviced was producing or shut down, should accompany the symbol.
4. Time spent on necessary work which is done in behalf of all or a group of machines should be recorded at the bottom of the time study observation sheet. These occasions should be rated whenever possible.
5. Note the time the study ends and count the production of each product for the period covered by the time study.
6. Normalize and average the rated walking occasions to determine the average normal walking time per servicing.
7. Determine the frequency per unit for each type of servicing by

dividing the number of units produced into the number of occasions of each type of servicing.

8. Compute the normal servicing time for each type of service by normalizing and averaging the rated actual time for the service in question. Add the average normal walking time per servicing to the normal servicing time for each type of service.
9. Increase the normal servicing and walking times by the appropriate fatigue allowance.
10. Multiply the individual servicing frequencies per unit by the respective values in Step 9 above to arrive at the servicing, walking and fatigue time per unit.
11. Total the various servicing, walking and fatigue values per unit applying to each product.
12. Determine the basic incentive standard in terms of hours per thousand units by substituting in the left-hand formula on page 39.
13. Determine the automatic run time, R, per unit for the product in question. Since most machines run at fixed speeds, the R values are usually pre-determined.
14. Determine the average normal servicing and walking time per unit during which the machine, on which the product in question is being produced, is producing.
15. Substitute in the right-hand formula on page 39.

CONCLUSIONS

The wage incentive plan for multiple machine assignments presented in this thesis has been designed to provide for consistently fair pay to the operator, in spite of the ever-changing conditions characteristic of this type of work. Experiments with the machine interference computer have proved that the mathematical solution of the machine interference problem is valid for both uniform and non-uniform conditions.

Whether the servicing requirements of the individual machines in a given assignment are the same or significantly different, the average idleness per machine due to interference is, for all practical purposes concerning wage incentive payment, the same as long as the average servicing requirements per machine are the same. Consequently, the operator's unavoidable idle time for a given total normal work load and number of machines is the same, regardless of the combinations of products manufactured.

The problem of varying work loads is solved in Figure 9. Case Problem No. 3 illustrates how the pay clerk rapidly approximates, through the use of Figure 9, the extent of unavoidable idle time inherent in a given assignment.

The matter of variable work loads is the crux of the problem of wage incentive payment for multiple machine assignments; product incentive standards for one total normal work load and number of machines are invalid for another total normal work load and number of machines. This problem has been overcome in the wage incentive plan presented in this thesis.

In the proposed plan, each product to be manufactured in multiple would have an individual incentive standard. In establishing the standards the time study man would assume that each machine is to be tended individually. By having the incentive rates stated on the individual attention basis it will be possible for the pay clerk to accurately compute the employees "working time" earnings. Then, by totaling the individual normal servicing loads for each product on each machine in the assignment the pay clerk can evaluate the operator's total normal work load. Quick reference to Figure 9 will tell the pay clerk approximately how much unavoidable idle time, if any, would have been experienced by the average operator under the conditions of the assignment. This idleness is then compensated for on the basis of the operator's guaranteed hourly pay rate.

Although it would be impractical to attempt to teach multiple machine operators the mathematical truth of Figure 9, the use of the curves in computing earnings should be taught to all. The operators can then figure their earnings as a check against the pay clerk. There is every reason to believe that the consistently equitable compensation features of the recommended wage incentive plan will win the approval of multiple machine operators who are so often "victims of circumstance" due to haphazard incentive rate setting methods. The plan is therefore recommended for general use as a consistently fair bases of wage incentive payment for the ever-changing conditions of multiple machine assignments.

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APPENDIX I

SAMPLE CALCULATIONS

APPENDIX I

SAMPLE CALCULATIONS

The calculations for a typical interference computer test are as follows:

INTERFERENCE TEST NO. 23 8/7/48

Mach.	% Block	Distrib. of Blocks	Beg. Rdg.	End. Rdg.	Diff.
1	0	-	-	-	-
2	15	5 - 5 - 5	2115.7	2125.8	10.1
3	15	15	1857.6	1867.0	9.4
4	10	5 - 5	1956.7	1966.5	9.8
5	0	-	-	-	-
6	20	5 - 5 - 5 - 5	1975.0	1983.7	8.7
7	15	5 - 5 - 5	2021.8	2030.7	8.9
8	0	-	-	-	-
9	15	10 - 5	2055.1	2065.1	10.0
10	5	5	1864.2	1874.6	10.4

Timer: Beg. 1274.80 End 1301.75
Operator's work time = 26.95 min.
Length of test = 30.00 min.
% Worked time = 26.95/30.00 = 90%
% Ave. serv. time/mach. = 90%/7
= 12.9%

Total revolutions	67.3
Number of machines	7
Ave. revs. per machine	9.61
Revs. with no interference	11.20

Ave. % inter./mach. $= \dfrac{11.20-9.61}{11.20}$
= 14.2%

% Ave. serv. time/mach. on individ. att'n basis = 12.9%/(100%-14.2%) = 15.0%

APPENDIX II

TABLES

TABLE I. Binomial Coefficients for n Values of 1 to 10

n	Binomial Coefficients										
1	1										
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	256	210	120	45	10	1

TABLE II. INTERFERENCE COMPUTER TEST RESULTS

Test No.	No.Machs.	Average %Serv.Time	Act.%Inter. On Computer	Jones Math.Value	% Deviation From Actual	Pinkerton Math.Value	% Deviation From Actual
1	8	12.1	13.8	11.0	-2.8	4.0	-9.8
2	8	12.6	13.4	12.4	-1.0	4.2	-9.2
3	8	13.6	13.8	15.4	1.6	4.9	-8.9
4	8	13.6	14.7	15.4	0.7	4.9	-9.8
5	8	16.5	26.8	26.6	-0.2	6.5	-21.3
6	8	8.9	4.5	4.9	0.4	2.3	-2.2
7	8	14.5	18.9	18.4	-0.5	5.3	-13.6
8	8	14.2	16.9	17.3	0.4	5.2	-11.7
9	8	14.3	16.8	17.7	0.9	5.2	-11.6
10	6	18.0	15.1	16.5	1.4	6.0	-9.1
11	10	11.5	16.3	16.7	0.4	4.3	-12.0
12	10	11.2	15.6	15.4	-0.2	4.3	-11.3
13	10	11.1	12.1	15.0	2.9	4.2	-7.9
14	10	12.7	21.1	23.0	1.9	5.0	-16.1
15	10	11.2	14.1	15.5	1.4	4.4	-9.7
16	9	10.4	9.8	9.4	-0.4	3.3	-6.5
17	9	11.6	11.6	12.7	1.1	4.0	-7.6
18	9	10.4	10.5	9.4	-1.1	3.3	-7.2
19	9	10.2	8.8	8.9	-0.1	3.2	-5.6
20	9	9.0	6.0	6.3	-0.3	2.5	-3.5
21	7	16.0	18.3	17.4	-0.9	5.5	-12.8
22	8	15.6	21.6	22.6	1.0	6.0	-15.6
23	7	15.0	14.2	14.8	0.6	5.0	-9.2
24	6	16.5	13.0	13.3	0.3	5.2	-9.8
25	6	18.5	14.7	17.4	2.7	6.4	-8.3
26	7	16.5	19.1	18.9	-0.2	5.9	-13.2
27	7	14.0	11.8	12.4	-0.6	4.5	-7.3
28	7	11.2	7.1	6.8	0.3	3.0	-4.1
29	8	10.1	7.6	6.8	1.8	2.9	-4.7
30	8	13.2	13.1	14.1	-1.0	4.5	-8.6
31	10	11.6	17.9	17.2	0.7	4.4	-13.5

TABLE II. INTERFERENCE COMPUTER TEST RESULTS CONT'D

Test No.	No.Machs.	Average %Serv.Time	Act.%Inter. On Computer	Jones Math.Value	% Deviation From Actual	Pinkerton Math.Value	% Deviation From Actual
32	10	9.8	8.8	10.3	-1.5	3.2	-5.6
33	10	8.8	6.5	7.6	-1.1	2.6	-3.9
34	10	9.9	9.2	10.6	-1.4	3.2	-6.0
35	9	10.4	8.7	9.4	-0.7	3.4	-5.3
36	10	11.1	15.0	15.0	0.0	4.0	-11.0
37	10	13.9	30.0	30.8	-0.8	5.8	-24.2
38	10	5.1	1.9	1.5	0.4	1.0	-0.9
39	9	6.9	2.9	3.1	-0.2	1.5	-1.4
40	9	12.9	16.4	17.7	-1.3	4.8	-11.6
41	9	12.9	19.5	17.7	1.8	4.8	-14.7
42	9	14.1	23.1	21.7	1.4	5.6	-17.5
43	8	16.0	24.4	24.2	0.2	6.2	-18.2
44	6	16.3	12.5	12.9	-0.4	5.2	-7.3
45	6	18.3	18.6	17.1	1.5	6.2	-12.4
46	6	16.5	12.5	13.3	-0.8	5.3	-7.2
47	6	14.0	9.1	9.0	0.1	4.0	-5.1
48	6	9.7	2.4	3.7	-1.3	2.1	-0.3
49	7	9.3	4.4	4.3	0.1	2.2	-2.2
50	5	14.4	5.2	6.7	-1.5	2.7	-2.5
51	5	19.1	11.6	12.8	-0.8	5.7	-5.9
52	5	11.6	3.9	4.1	-0.2	2.4	-1.5
53	5	16.2	7.3	8.8	-1.5	4.3	-3.0
54	4	19.0	8.7	8.4	0.3	4.5	-4.2
55	4	18.4	8.4	7.8	0.6	4.1	-4.3
56	4	16.6	6.4	6.2	0.2	3.6	-2.8
57	4	15.7	5.5	5.4	0.1	3.2	-2.3
58	4	12.4	1.8	3.2	-1.4	2.0	0.2
59	5	11.6	3.5	4.2	-0.7	2.4	-1.1
60	6	12.5	6.1	6.8	-0.7	3.3	-2.8
61	7	10.9	6.6	6.4	0.2	2.9	-3.7
62	8	11.8	9.7	10.3	-0.6	3.8	-5.9

TABLE III. MULTIPLE MACHINE ASSIGNMENT OPERATOR'S PROBABLE EARNINGS

FOR VARIOUS CONDITIONS WHEN TENDING GROUPS OF 4 MACHINES

% Normal Work Load	%Unavoid. Idle Time (or U)			Probable Percentage Incentive Earnings								
				No Allow. for U			100% Allow. for U			125% Allow. for U		
	100%E	125%E	150%E	100%E	125%E	150%E	100%E	125%E	150%E	100%E	125%E	150%E
100	16	25	35	*84	*94	*98	100	119	133	104	125	142
110	12	20	29	*88	100	107	100	120	136	103	125	143
120	8	15	24	*92	106	114	100	121	138	102	125	144
130	6	11	19	*94	111	121	100	122	140	102	125	145
140	4	8	15	*96	115	127	100	123	142	101	125	146
150	2	6	12	*98	118	132	100	124	144	101	125	147
160	1	3	9	*99	121	137	100	124	146	100	125	148
170		2	6	100	123	141	100	125	147	100	125	149
180		1	4	100	124	144	100	125	148	100	125	150

* Denotes those occasions for which the operator would be granted 100% base pay in spite of the less than 100% efficiency.

TABLE IV. MULTIPLE MACHINE ASSIGNMENT OPERATOR'S PROBABLE EARNINGS

FOR VARIOUS CONDITIONS WHEN TENDING GROUPS OF 10 MACHINES

% Normal Work Load	% Unavoid. Idle Time (or U)			Probable Percentage Incentive Earnings								
				No. Allow. for U			100% Allow. for U			125% Allow. for U		
	100%E	125%E	150%E	100%E	125%E	150%E	100%E	125%E	150%E	100%E	125%E	150%E
100	12	23	34	*88	*96	*99	100	119	133	103	125	142
110	7	17	28	*93	104	108	100	121	136	102	125	143
120	3	12	22	*97	110	117	100	122	139	101	125	144
130	1	8	17	*99	115	124	100	123	141	100	125	145
140		4	13	100	120	131	100	124	144	100	125	147
150		1	9	100	124	137	100	125	146	100	125	148
160			6	100	125	141	100	125	147	100	125	149
170			3	100	125	146	100	125	149	100	125	150
180			1	100	125	149	100	125	150	100	125	150

* Denotes those occasions for which the operator would be granted 100% base pay in spite of the less than 100% efficiency.

TABLE V. MULTIPLE MACHINE ASSIGNMENT OPERATOR'S PROBABLE EARNINGS

FOR VARIOUS CONDITIONS WHEN TENDING GROUPS OF 30 MACHINES

% Normal Work Load	%Unavoid.Idle Time(or U)			Probable Percentage Incentive Earnings								
				No Allow. for U			100% Allow. for U			125% Allow. for U		
	100%E	125%E	150%E	100%E	125%E	150%E	100%E	125%E	150%E	100%E	125%E	150%E
100	7	22	34	*93	*98	*99	100	120	133	102	125	143
110	1	16	27	*99	105	110	100	121	138	100	125	144
120		10	21	100	113	119	100	123	140	100	125	145
130		5	15	100	119	128	100	124	143	100	125	147
140		1	10	100	124	135	100	125	145	100	125	148
150			6	100	125	141	100	125	147	100	125	149
160			2	100	125	147	100	125	149	100	125	150
170				100	125	150	100	125	150	100	125	150
180				100	125	150	100	125	150	100	125	150

* Denotes those occasions for which the operator would be granted 100% base pay in spite of the less than 100% efficiency.

TABLE VI. MULTIPLE MACHINE ASSIGNMENT OPERATOR'S PROBABLE EARNINGS

FOR VARIOUS CONDITIONS WHEN TENDING GROUPS OF 100 MACHINES

% Normal Work Load	%Unavoid. Idle Time (or U)			Probable Percentage Incentive Earnings								
				No Allow. for U			100% Allow. for U			125% Allow. for U		
	100%E	125%E	150%E	100%E	125%E	150%E	100%E	125%E	150%E	100%E	125%E	150%E
100	2	20	33	*98	100	101	100	120	134	101	125	144
110		13	26	100	109	111	100	122	137	100	125	145
120		5	20	100	119	120	100	124	140	100	125	146
130			14	100	125	129	100	125	143	100	125	147
140			8	100	125	138	100	125	146	100	125	148
150			2	100	125	147	100	125	149	100	125	149
160				100	125	150	100	125	150	100	125	150
170				100	125	150	100	125	150	100	125	150
180				100	125	150	100	125	150	100	125	150

* Denotes those occasions for which the operator would be granted 100% base pay in spite of the less than 100% efficiency.

APPENDIX III

FIGURES

FIGURE I.
A TYPICAL MULTIPLE MACHINE ASSIGNMENT

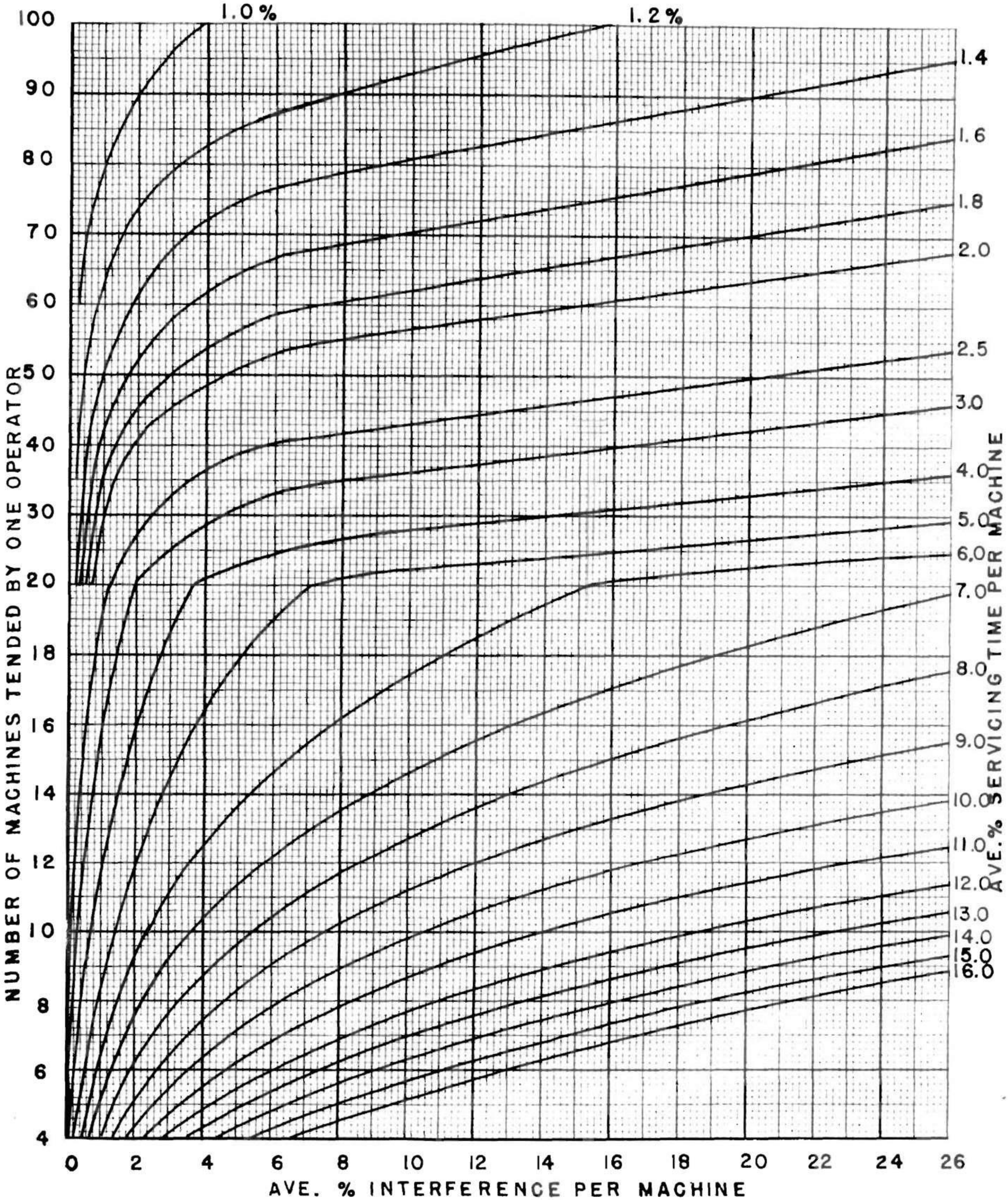

FIGURE 2. MACHINE INTERFERENCE VERSUS SERVICING TIME FOR GROUPS OF 4 TO 100 MACHINES

FIGURE 3.
INTERFERENCE COMPUTER SET UP FOR TIME STUDY PRACTICE IN LABORATORY

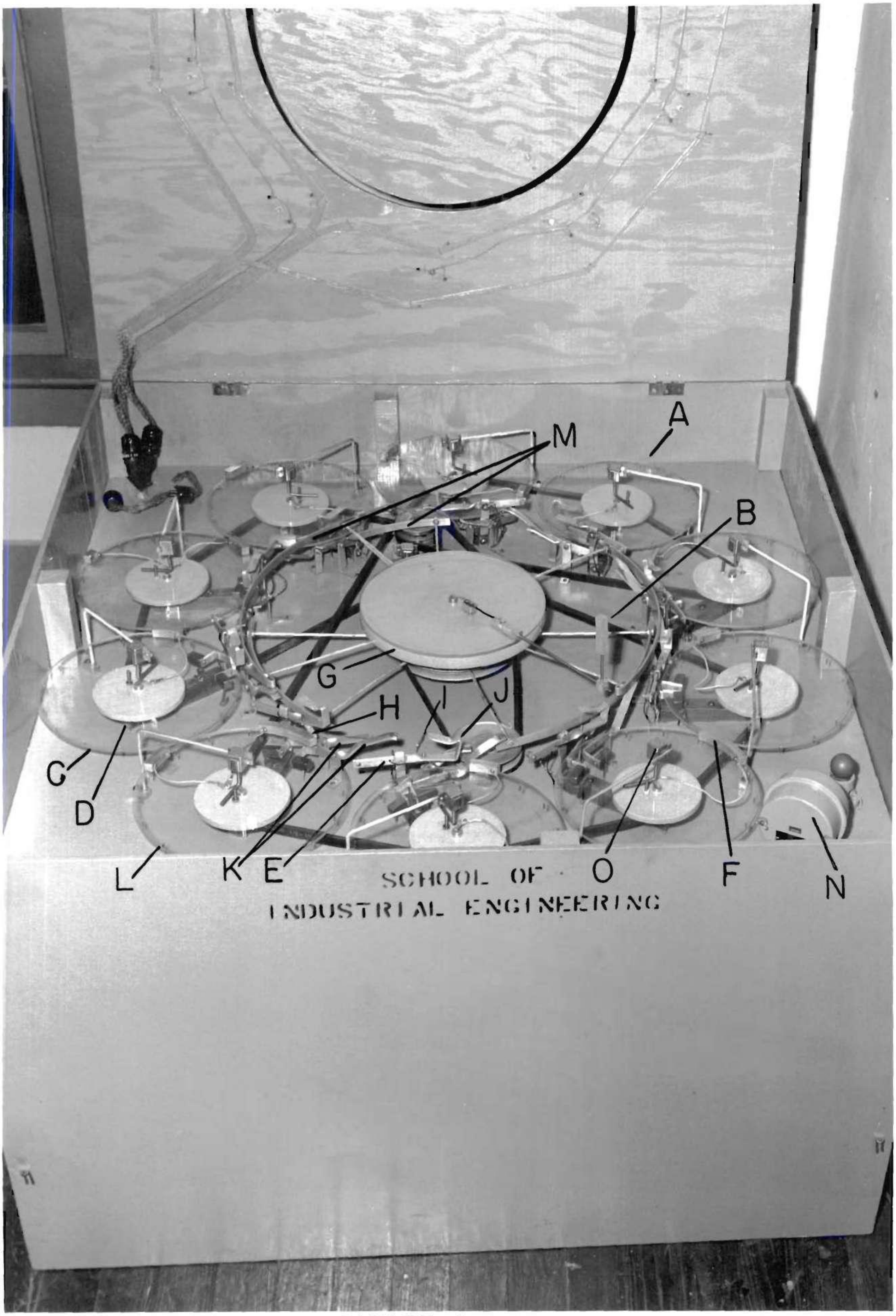

FIGURE 4.
A TYPICAL EXPERIMENT UNDER WAY

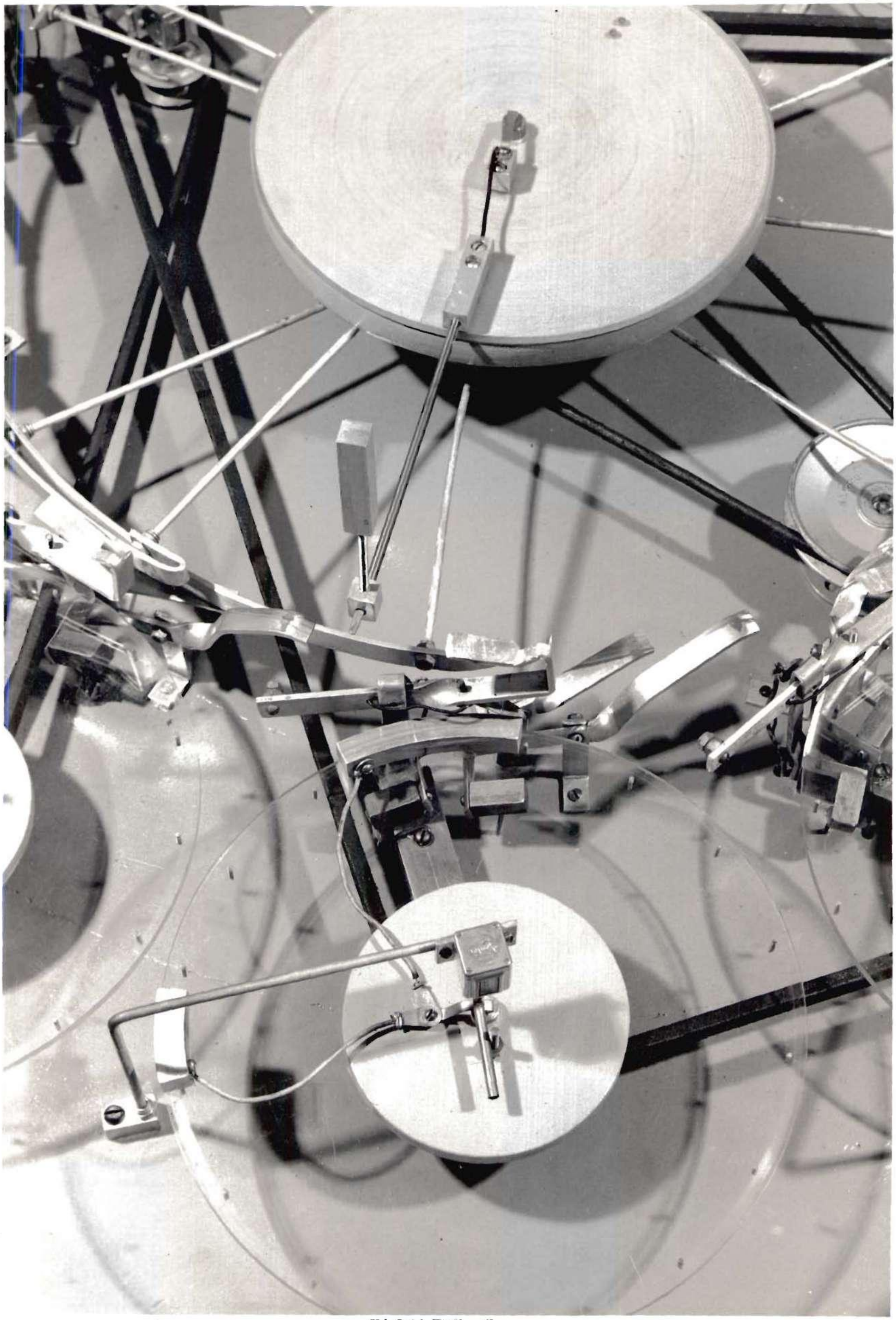

FIGURE 5.
CLOSE UP VIEW OF INTERFERENCE COMPUTER

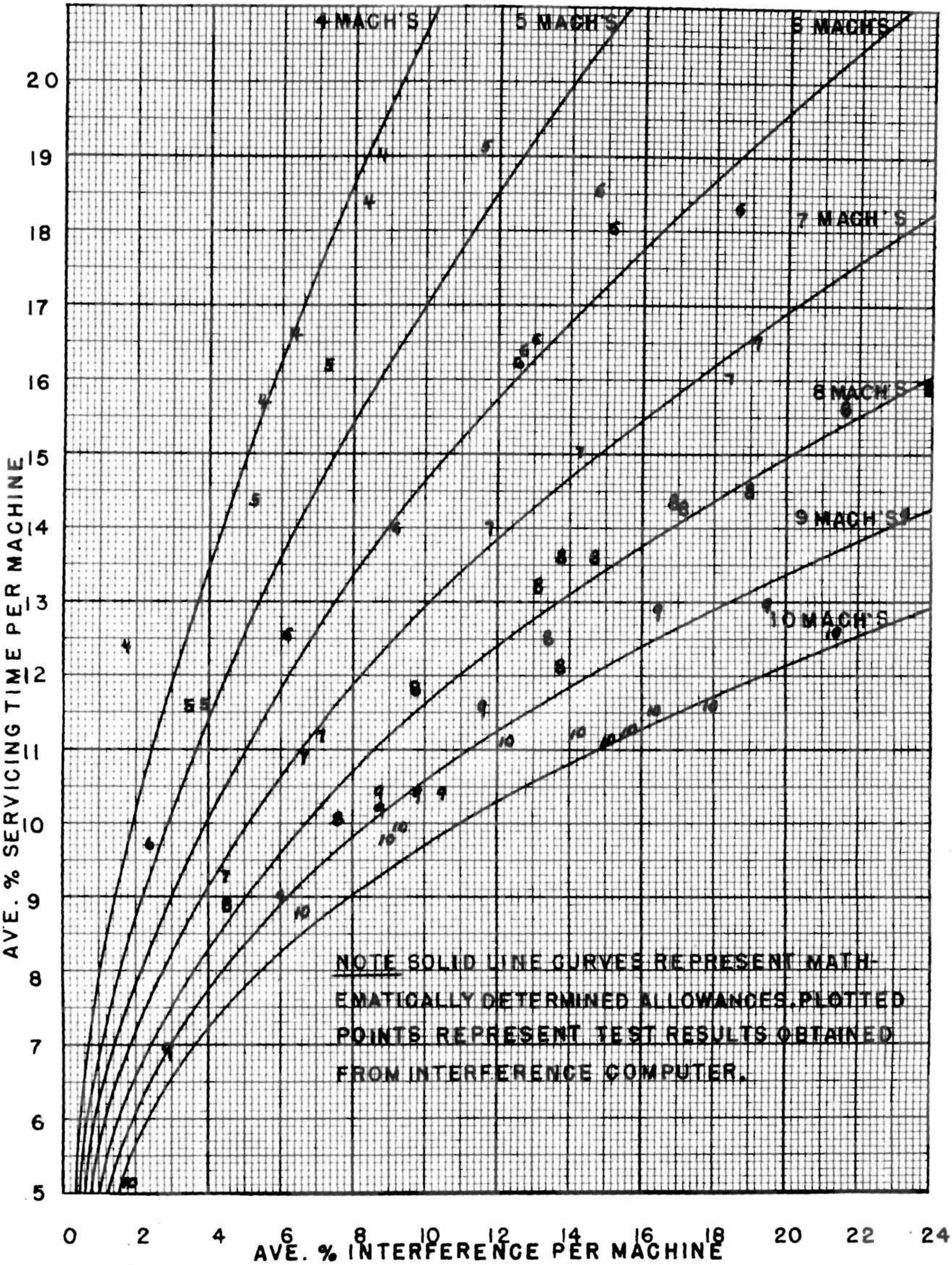

FIGURE 6. MACHINE INTERFERENCE VERSUS SERVICING TIME FOR GROUPS OF 4 TO 10 MACHINES

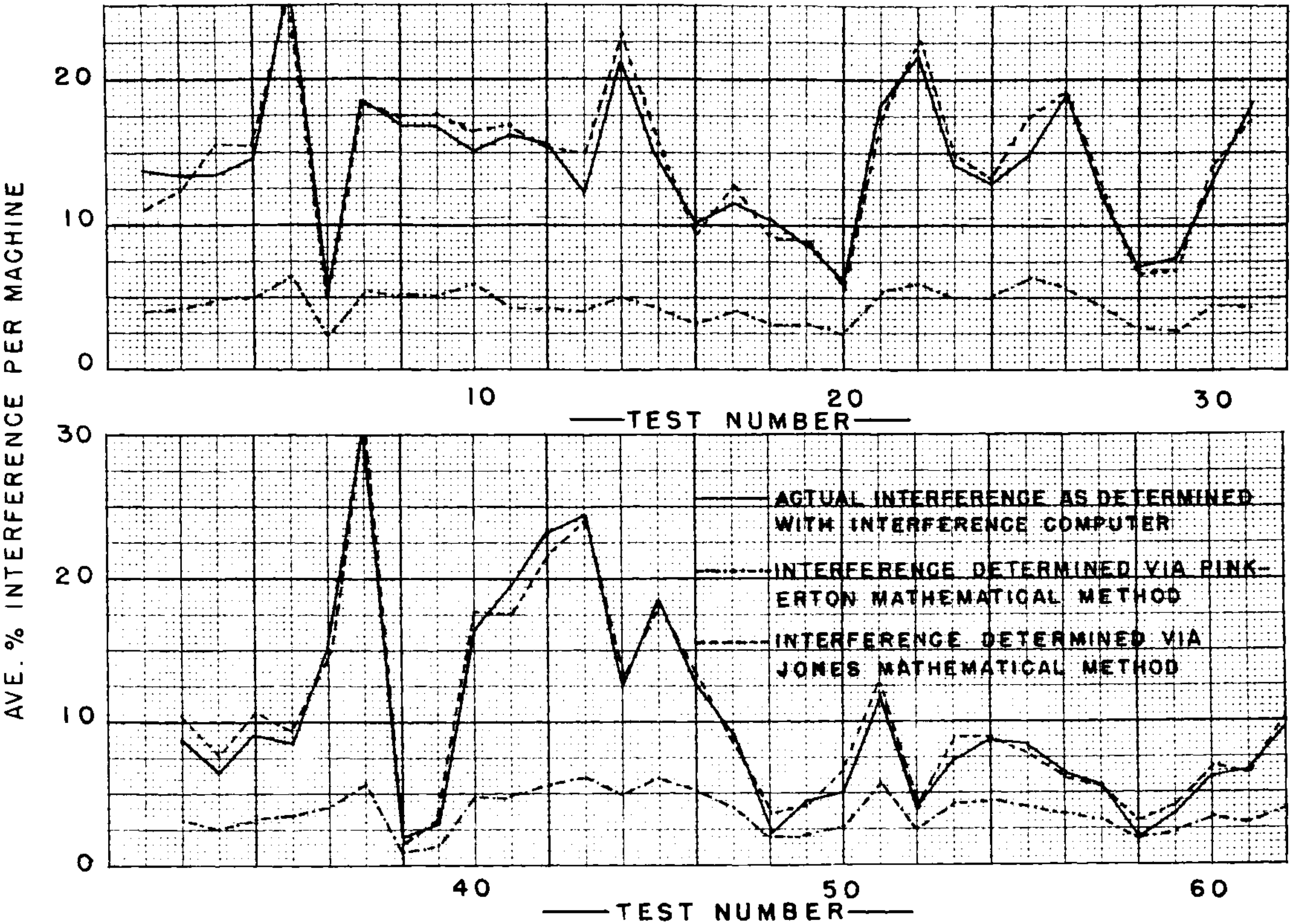

FIGURE 7. COMPARISON OF INTERFERENCE COMPUTER RESULTS WITH MATHEMATICALLY DETERMINED INTERFERENCE

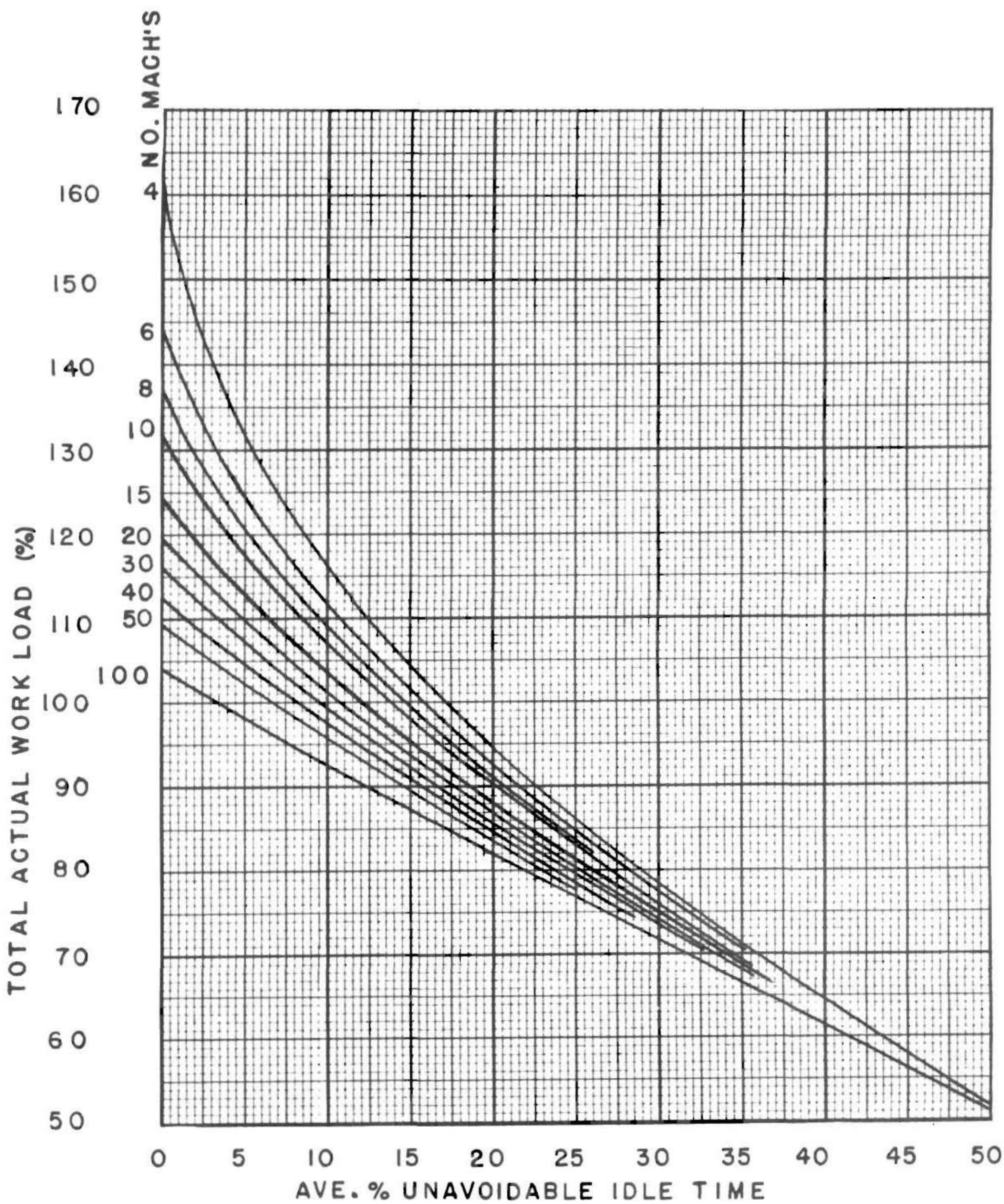

FIGURE.8. UNAVOIDABLE IDLE TIME VERSUS ACTUAL WORK LOAD WHEN ONE OPERATOR TENDS GROUPS OF 4 TO 100 MACHINES

106089

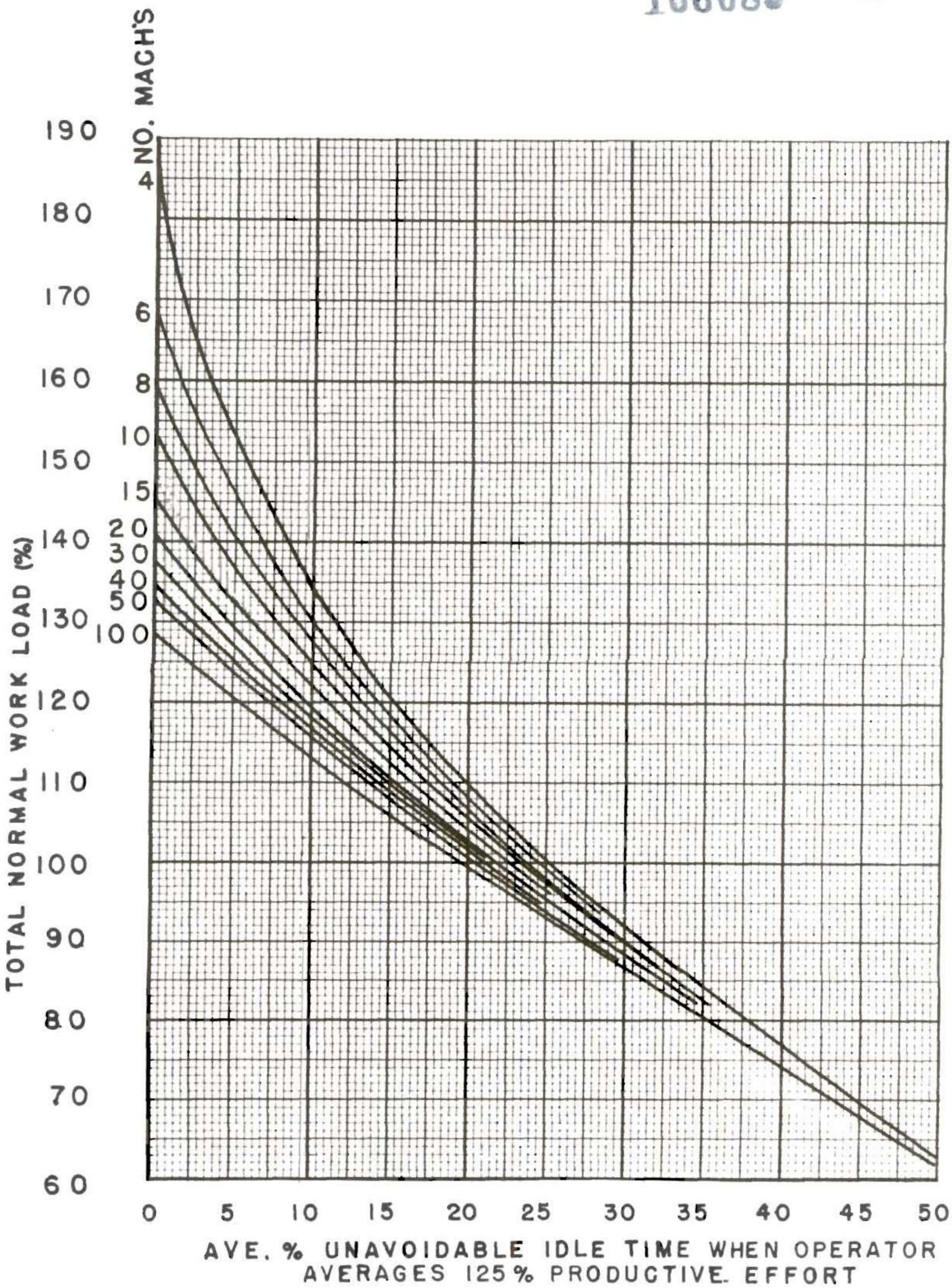

FIGURE 9. UNAVOIDABLE IDLE TIME VERSUS ACTUAL WORK LOAD WHEN ONE OPERATOR AVERAGES 125% PRODUCTIVE EFFORT IN TENDING GROUPS OF 4 TO 100 MACHINES